CÁLCULO 4

1ª LISTA DE EXERCÍCIOS

Transformada de Laplace

1. Calcule a transformada de Laplace da função dada usando a definição:

a) $f(t) = 3$ b) $f(t) = 2t$ c) $f(t) = e^{2t}$

2. Determine a transformada de Laplace da função dada:

a) $\cos t - \operatorname{sen} t$ b) $4t \operatorname{sen} 2t$
c) $t^2 - 3t + 5$ d) $t - \cos 5t$
e) $2t^2 e^{-3t} - 4t + 1$ f) $(t + 4)^2$
g) $3e^{-t} + \operatorname{sen} 6t$ h) $t^3 - 3t + \cos 4t$
i) $-3 \cos 2t + 5 \operatorname{sen} 4t$

3. Calcule a transformada inversa de Laplace:

a) $\dfrac{-2}{s+16}$ b) $\dfrac{2s-5}{s^2+16}$
c) $\dfrac{3}{s-7} + \dfrac{1}{s^2}$ d) $\dfrac{5}{(s+7)^2}$
e) $\dfrac{1}{s-4} - \dfrac{6}{(s-4)^2}$ f) $\dfrac{2}{s^4}\left[\dfrac{1}{s} - \dfrac{3}{s^2} - \dfrac{4}{s^6}\right]$

4. Use a transformada de Laplace para resolver os seguintes problemas de valor inicial:

a) $y' + 4y = 1$; $y(0) = -3$ b) $y' + 4y = \cos t$; $y(0) = 0$
c) $y' + 2y = e^{-t}$; $y(0) = 1$ d) $y'' + y = 1$; $y(0) = 6$, $y'(0) = 0$

5. A partir da decomposição em frações racionais, verifique que:

$$L^{-1}\left[\frac{s+1}{s(s^2+1)}\right] = 1 + \operatorname{sen} t - \cos t$$

6. Resolva as equações diferenciais:

a) $y' + y = e^{2t}$; $y(0) = 0$
b) $y'' - 4y' + 4y = 0$; $y(0) = 0$, $y'(0) = 1$
c) $y'' + 4y = 0$; $y(0) = 1$, $y(\frac{\pi}{4}) = -1$

7. Calcule as transformadas de Laplace das funções dadas usando os teoremas de deslocamento.

a) $(t^3 - 3t + 2)\, e^{-2t}$ b) $e^{-3t}\,(t - 2)$
c) $e^{4t}\,(1 - \cos t)$ d) $e^{-t}\,(1 - t^2 + \operatorname{sen} t)$

8. Determine a transformada inversa de Laplace:

a) $\dfrac{1}{s^2 + 4s + 12}$ b) $\dfrac{1}{s^2 - 4s + 5}$

c) $\dfrac{1}{s^3} e^{-5s}$ d) $\dfrac{s}{s^2 + 9} e^{-2s}$

e) $\dfrac{3}{s + 2} e^{-4s}$ f) $\dfrac{s - 4}{s^2 - 8s + 10}$

g) $\dfrac{1}{(s - 5)^3} e^{-s}$ h) $\dfrac{1}{s^2 + 6s + 7}$

i) $\dfrac{2s + 4}{s^2 - 4s + 4}$

9. Resolva o problema de valor inicial:

$y'' + 4y = f(t)$ $y(0) = 1,\ y'(0) = 0$

$$\begin{cases} f(t) = 0, & 0 \le t < 4 \\ f(t) = 3, & t \ge 4 \end{cases}$$

10. Resolva os problemas de valor inicial:

a) $y'' - 5y' + 6y = f(t)$ $y(0) = 0,\ y'(0) = 0$
b) $y'' + 9y = f(t)$ $y(0) = -1,\ y'(0) = 1$

Respostas

1. Ver a resolução de exemplos da sala de aula.

2.

a) $\dfrac{s-1}{s^2+1}$

b) $\dfrac{16s}{(s^2+4)^2}$

c) $\dfrac{2}{s^3}-\dfrac{3}{s^2}+\dfrac{5}{s}$

d) $\dfrac{1}{s^2}-\dfrac{s}{s^2+25}$

e) $\dfrac{4}{(s+3)^3}-\dfrac{4}{s^2}+\dfrac{1}{s}$

f) $\dfrac{2}{s^3}+\dfrac{8}{s^2}+\dfrac{16}{s}$

g) $\dfrac{3}{s+1}+\dfrac{6}{s^2+36}$

h) $\dfrac{6}{s^4}-\dfrac{3}{s^2}+\dfrac{s}{s^2+16}$

i) $-\dfrac{3s}{s^2+4}+\dfrac{20}{s^2+16}$

3.

a) $-2\,e^{-16t}$

b) $2\cos 4t-\dfrac{5}{4}\,\text{sen}\,4t$

c) $3\,e^{7t}+t$

d) $5\,t\,e^{-7t}$

e) $e^{4t}-6\,t\,e^{4t}$

f) $\dfrac{1}{12}t^4-\dfrac{6}{5!}t^5-\dfrac{8}{9!}t^9$

4.

a) $y=\dfrac{1}{4}-\dfrac{13}{4}e^{-4t}$

b) $y=-\dfrac{4}{17}e^{-4t}+\dfrac{4}{17}\cos t+\dfrac{1}{17}\,\text{sen}\,t$ (*)

c) $y=e^{-t}$

d) $y=1+5\cos t$

(*) Sugestão: Tente $\dfrac{A}{s+4}+\dfrac{B}{s^2+1}+\dfrac{Cs}{s^2+1}$

6.

a) $y(t)=-\dfrac{1}{3}e^{-t}+\dfrac{1}{3}e^{2t}$

b) $y(t)=t\,e^{2t}$

c) $y(t)=\cos 2t-\text{sen}\,2t$; $y'(0)=-2$ (**)

(**) Sugestão: Trate $y'(0)$ como um parâmetro desconhecido e determine-o ao fim do processo.

7.

a) $\dfrac{6}{(s+2)^4} - \dfrac{3}{(s+2)^2} + \dfrac{2}{s+2}$

b) $\dfrac{1}{(s+3)^2} - \dfrac{2}{s+3}$

c) $\dfrac{1}{s-4} - \dfrac{s-4}{s^2-8s+17}$

d) $\dfrac{1}{s+1} - \dfrac{2}{(s+1)^3} + \dfrac{1}{(s+1)^2+1}$

8.

a) $\dfrac{1}{\sqrt{8}} e^{-2t} \operatorname{sen} \sqrt{8}t$

b) $e^{2t} \operatorname{sen} t$

c) $\dfrac{1}{2}(t-5)^2 H(t-5)$

d) $H(t-2)\cos 3(t-2)$

e) $3e^{-2(t-4)} H(t-4)$

f) $\mathrm{e}^{4t} \cosh \sqrt{6}\, t$

g) $\dfrac{1}{2}(t-1)^2 e^{5(t-1)} H(t-1)$

h) $\dfrac{1}{\sqrt{2}} e^{-3t} \operatorname{senh} \sqrt{2}\, t$

i) $2\mathrm{e}^{2t} + 8\, t\, e^{2t}$

9.

$y(t) = \cos 2t + \dfrac{3}{4}\left[1 - \cos 2(t-4) H(t-4)\right]$

10.

a) $y(t) = e^{3t} * f(t) - e^{2t} * f(t)$

b) $y(t) = \dfrac{1}{3} \operatorname{sen} 3t * f(t) - \cos 3t + \dfrac{1}{3} \operatorname{sen} 3t$