

① Pipa Jalur Permanen

$$\dot{m} = 50 \text{ kg/s}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$\bar{v}_{1,2} = ?$$

$$Q = ?$$

$$d_1 = 0,20 \text{ m}$$

$$d_2 = 0,10 \text{ m}$$

sol:  $\dot{m} = \rho \cdot A \cdot \bar{v}$   
 $50 = 1000 \cdot Q$

$$\rightarrow Q = 0,05 \text{ m}^3/\text{s}$$

$$Q_1 = \bar{v}_1 \cdot A_1$$

$$0,05 = \bar{v}_1 \cdot \frac{\pi \cdot 0,2^2}{4}$$

$$\rightarrow \bar{v}_1 = 1,6 \text{ m/s}$$

$$Q = \bar{v}_2 \cdot A_2$$

$$0,05 = \bar{v}_2 \cdot \frac{\pi \cdot 0,1^2}{4}$$

$$\rightarrow \bar{v}_2 = 6,4 \text{ m/s}$$

② Bernoulli antara A & C

$$\frac{P_A}{\rho} + z_A + \frac{v_A^2}{2g} = \frac{P_C}{\rho} + z_C + \frac{v_C^2}{2g}$$

$$\left\{ \begin{array}{l} z_C = 0 \text{ (NR)} \\ v_A > v_C \rightarrow v_A \approx 0 \end{array} \right.$$

$$P_C = P_{atm} \rightarrow P_A = P_{atm} \rightarrow P_{atm} = 0$$

$$200 \text{ mmHg} - \rho$$

$$700 \text{ mmHg} - 101325 \text{ Pa}$$

$$\rightarrow \rho = 20664,47 \text{ Pa}$$

$$\frac{20664,47}{1010} + 8 = \frac{v_C^2}{2 \cdot 9,81}$$

$$20,728 \cdot 19,62 = v_C^2$$

$$v_C = 14,5 \text{ m/s}$$

$$Q = A \cdot v$$

$$Q = \frac{\pi \cdot 0,04^2}{4} \cdot 14,5 \rightarrow \boxed{Q = 0,028 \text{ m}^3/\text{s}}$$

Bernoulli entre A e B

$$\frac{P_A}{\rho} + z_A + \frac{v_A^2}{2g} = \frac{P_B}{\rho} + z_B + \frac{v_B^2}{2g} \quad \left\{ \begin{array}{l} v_A \gg v_B \rightarrow v_A = 0 \\ v_B = v_C \end{array} \right.$$

$$\frac{20664,42}{9810} + 0 = \frac{P_B}{9810} + 4 + \frac{14,5^2}{2 \cdot 9,81}$$

$$2,108 = \frac{P_B}{9810} + 14,716$$

$$\boxed{P_B = -39221,42 \text{ Pa}}$$

$$\boxed{P_{abs} = 62103,58 \text{ Pa}}$$

4) Bernoulli entre A e B

$$\frac{P_A}{\rho} + z_A + \frac{v_A^2}{2g} = \frac{P_B}{\rho} + z_B + \frac{v_B^2}{2g}$$

parâmetros:

$$P_A = P_B + \cancel{\rho \cdot H_{H_2O}} + 0,3 \cdot \rho_{Hg} - 0,3 \cdot \rho_{H_2O} - \cancel{\rho \cdot H_{H_2O}} - z_A \cdot \rho_{H_2O}$$

$$P_A - P_B = 0,3 \cdot \rho_{Hg} - 0,3 \cdot \rho_{H_2O} - z_A \cdot \rho_{H_2O}$$

$$Q = A \cdot v \rightarrow v = \frac{Q \cdot 4}{\pi \cdot d^2}$$

$$\frac{P_A - P_B}{\rho_{H_2O}} + z_A + \frac{Q^2 \cdot 4^2}{\pi^2 \cdot d_A^2} \cdot \frac{1}{2g} = \frac{Q^2 \cdot 4^2}{\pi^2 \cdot d_B^2} \cdot \frac{1}{2g}$$

(2.)

$$\left( \frac{0,3 \cdot \delta_{Hg} - 0,3 \cdot \delta_{H_2O} - 3 \delta_{H_2O}}{\delta_{H_2O}} \right) + z_A = \left( \frac{Q^2 \cdot 16}{\pi^2 \cdot 0,05^4} - \frac{Q^2 \cdot 16}{\pi^2 \cdot 0,08^4} \right) \cdot \frac{1}{2g}$$

$$0,3 \cdot (13,6 \cdot 9810) - 0,3 \cdot 9810 = (259645,4 Q^2 - 39618,45 Q^2) \cdot \frac{1}{2g} \cdot 9810$$

$$\frac{1}{9810} \cdot 37081,8 \cdot 19,62 = 220020,7 Q^2$$

$$Q^2 = 3,3066 \cdot 10^{-4}$$

$$Q = 1,82 \cdot 10^{-2} \text{ m}^3/\text{s}$$

5) Bernoulli entre (1) e (2):

$$\frac{P_1}{\rho} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho} + z_2 + \frac{v_2^2}{2g}$$

$$\left. \begin{array}{l} P_1 = P_{atm} = 0 \\ v_1 \gg v_2 \rightarrow v_1 \approx 0 \end{array} \right\} z_2 = 0 \text{ (NR)}$$

$$3,66 = \frac{P_2}{9810} + \frac{v_2^2}{19,62}$$

Manometria:

$$P_2 = P_{atm} + 0,15 \cdot \delta_{Hg} - 0,61 \cdot \delta_{H_2O}$$

$$P_2 = 0,15 \cdot (13,6 \cdot 9810) - 0,61 \cdot 9810 \rightarrow P_2 = 14028,3 \text{ Pa}$$

$$3,66 = \frac{14028,3}{9810} + \frac{v_2^2}{19,62}$$

$$2,23 \cdot 19,62 = v_2^2$$

$$\rightarrow v_2 = 6,61 \text{ m/s}$$

$$Q = A_2 v_2$$

$$Q = \frac{\pi \cdot 0,05^2}{4} \cdot 6,61$$

$$\rightarrow Q = 13 \cdot 10^{-3} \text{ m}^3/\text{s}$$

$$Q = 13 \text{ L/s}$$

(3.)

3) Bernoulli entre ① e ②:

$$\frac{P_1}{\rho} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho} + z_2 + \frac{v_2^2}{2g}$$

$$\left\{ \begin{array}{l} z_2 = 0 \text{ (NR)} \\ v_1 \gg v_2 \rightarrow v_1 \approx 0 \end{array} \right. \quad P_2 = P_{\text{atm}} = 0$$

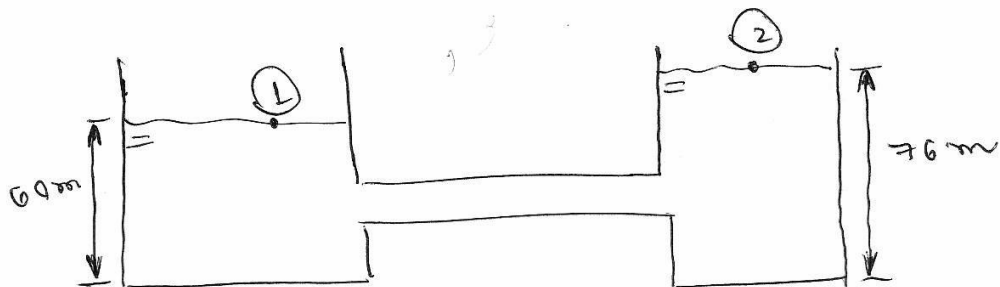
$$\frac{101325}{9810} + 5 = \frac{v_2^2}{19,62}$$

$$v_2^2 = 300,75$$

$$\hookrightarrow v_2 = 17,34 \text{ m/s}$$

6) Uma tubulação de ferro fundido ( $E = 2,5 \cdot 10^{-4} \text{ m}$ ) que tem  $0,3 \text{ m}$  de diâmetro e  $300 \text{ m}$  de comprimento comunica dois recipientes que tem seus níveis de superfícies a uma altura de  $60 \text{ m}$  e  $76 \text{ m}$ . Calcule a vazão através desta tubulação, supondo-se que a água se encontra a uma temperatura de  $10^\circ \text{C}$  ( $\nu = 1,3 \cdot 10^{-6} \text{ m}^2/\text{s}$ ) e que a entrada no tubo é normal.

Dados:  $K_e = 0,5$  (entrada e saída da tubulação)  
 $K_s = 1,0$



ap. silvares

Bernoulli entre ① e ②:

Sol. 
$$\frac{P_1}{\rho} + z_1 + \frac{v_1^2}{2g} + h_{p2-1} = \frac{P_2}{\rho} + z_2 + \frac{v_2^2}{2g}$$

$$\left\{ \begin{array}{l} P_1 = P_2 = P_{atm} \\ v_1 = v_2 = 0 \end{array} \right.$$

fluxo de ② p/ ①

$$60 + (h_d + h_l) = 76$$

$$h_d + h_l = 16$$

$$h_d = f \cdot \frac{l}{d} \cdot \frac{v^2}{2g}$$

$$h_l = h_{le} + h_{ls} = (K_e + K_s) \cdot \frac{v^2}{2g}$$

$$\left( f \cdot \frac{l}{d} + K_e + K_s \right) \cdot \frac{v^2}{2g} = 16$$

$$\left( f \cdot \frac{300}{0,3} + 0,5 + 1,0 \right) \cdot \frac{v^2}{19,62} = 16$$

$$v = \sqrt{\frac{313,92}{2000f + 1,5}}$$

Cálculo de  $f$ :

$$\boxed{f = 0,030} \rightarrow v = 3,16 \text{ m/s}$$

$$Re = \frac{v \cdot d}{\nu} = \frac{3,16 \cdot 0,3}{1,3 \cdot 10^{-6}} \rightarrow Re = 7,3 \cdot 10^5$$

$$\frac{D}{E} = \frac{0,3}{2,5 \cdot 10^{-4}} \rightarrow \frac{D}{E} = 1200$$

Diagrama Rouse-Woody:  $f = 9019$

Recalculando:

$$v = 3,91 \text{ m/s}$$

$$Re = \frac{3,91 \cdot 0,3}{1,31 \cdot 10^{-6}} \rightarrow Re = 9,02 \cdot 10^5$$

$$\frac{D}{E} = \frac{0,3}{2,5 \cdot 10^{-4}} \rightarrow \frac{D}{E} = 1200$$

(2.)

Diagrama Ramus.  $\nu = 0,019$

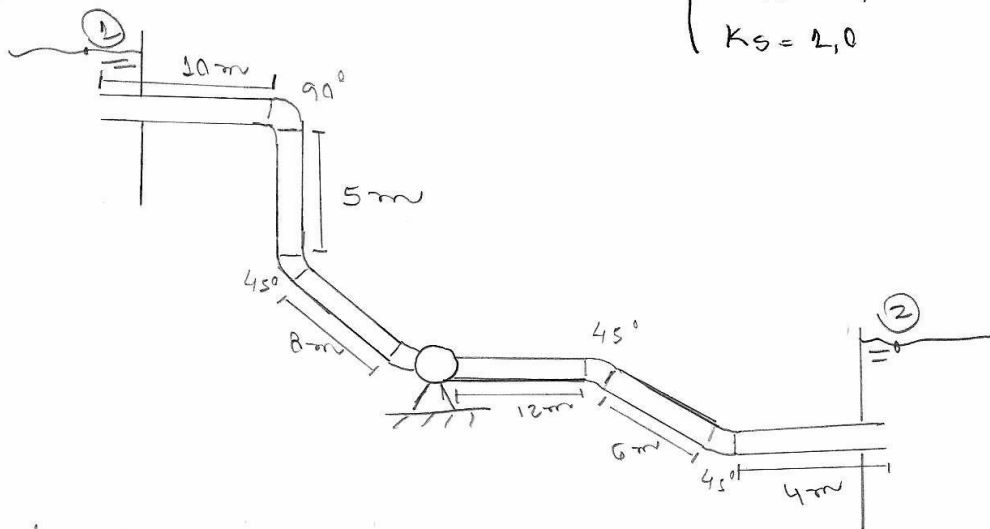
$$v = 3,91 \text{ m/s}$$

$$Q = A \cdot v = \frac{\pi \cdot 0,3^2}{4} \cdot 3,91 \rightarrow Q = 0,277 \text{ m}^3/\text{s}$$

7) Determinar a perda de carga da instalação abaixo e a vazão, sabendo-se que o seu diâmetro é de 0,5 m e que a velocidade é de 4 m/s.

Dados:  $E = 5 \cdot 10^{-5} \text{ m}$   
 $\nu = 1,3 \cdot 10^{-6} \text{ m}^2/\text{s}$

$$\left\{ \begin{array}{l} K_e = 1,0 \\ K_{90^\circ} = 0,4 \\ K_{45^\circ} = 0,2 \\ K_s = 2,0 \end{array} \right.$$



Sol:

$$h_{p1 \rightarrow 2} = h_d + h_l$$

$$h_l = K \cdot \frac{v^2}{2g} = (K_e + K_{90^\circ} + K_{45^\circ} \cdot 3 + K_s) \cdot \frac{v^2}{2g}$$

$$h_l = (1,0 + 0,4 + 3 \cdot 0,2 + 2,0) \cdot \frac{4^2}{19,62} \rightarrow h_l = 2,45 \text{ m}$$

(3.)

$$h_d = f \cdot \frac{l}{d} \cdot \frac{v^2}{2g} = f \cdot \frac{(10 + 5 + 8 + 12 + 6 + 4)}{0,5} \cdot \frac{4,2^2}{2 \cdot 9,81}$$

$$h_d = f \cdot 90 \cdot 0,82 \rightarrow \boxed{h_d = 73,8 \cdot f}$$

$$\left\{ \begin{array}{l} Re = \frac{v \cdot d}{\nu} = \frac{4,2 \cdot 0,5}{2,3 \cdot 10^{-6}} \rightarrow Re = 1,50 \cdot 10^6 \end{array} \right.$$

$$\frac{D}{\epsilon} = \frac{0,5}{5 \cdot 10^{-5}} \rightarrow \frac{D}{\epsilon} = 1 \cdot 10^4$$

Diagrama de Rouse - wlosdy:  $f = 0,0122$

$$h = 73,8 \cdot 0,0122 \rightarrow \boxed{h_d = 0,9000 \text{ m}}$$

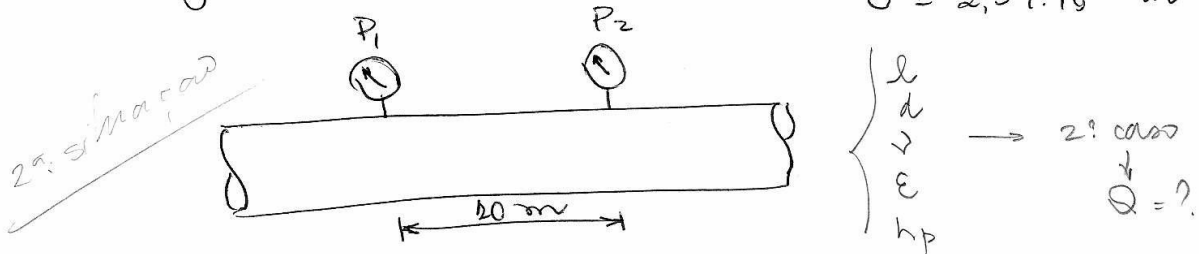
$$h_{p1 \rightarrow 2} = 2,45 + 0,9000$$

$$\boxed{h_{p1 \rightarrow 2} = 3,35 \text{ m}}$$

$$Q = A \cdot v = \frac{\pi \cdot 0,5^2}{4} \cdot 4 \rightarrow \boxed{Q = 0,785 \text{ m}^3/\text{s}}$$



8) Calcular a vazão de água num conduto de ferro fundido, sendo dados  $D = 20 \text{ cm}$ ,  $f = 0,7 \cdot 10^{-6} \frac{\text{m}^2}{\text{s}}$  e sabendo-se que dois manômetros instalados a uma distância de  $10 \text{ m}$  indicam, respectivamente,  $1,5 \text{ kgf/cm}^2$  e  $1,45 \text{ kgf/cm}^2$ . Dado:  $\gamma_{\text{H}_2\text{O}} = 10^3 \text{ kgf/m}^3$   
 $E = 2,59 \cdot 10^{-4} \text{ m}$



Sol: Bernoulli entre (1) e (2):

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_{p1 \rightarrow 2}$$

$$\begin{cases} z_1 = z_2 = NR \\ v_1 = v_2 \end{cases}$$

0,05

$$\frac{\text{kgf}}{\text{cm}^2} \cdot \frac{10}{100 \text{ cm}}$$

$$h_{p1 \rightarrow 2} = \frac{P_1 - P_2}{\gamma} = \frac{(1,5 - 1,45) \text{ kgf/cm}^2}{10^3 \frac{\text{kgf}}{\text{m}^3}} \cdot \frac{10^4 \text{ cm}^2}{1 \text{ m}^2} = 0,05 \text{ kgf} \cdot 10 = 0,5 \text{ kgf}$$

$$h_{p1 \rightarrow 2} = 0,5 \text{ m}$$

$$Q = A \cdot v = \frac{\pi \cdot d^2}{4} \cdot v \quad ?$$

$$\frac{\text{kgf}}{\text{cm}^2} \cdot \frac{\text{cm}^2}{\text{kgf}} \cdot \frac{1 \text{ m}}{100 \text{ cm}}$$

$$h_{p1 \rightarrow 2} = h_d = \frac{f \cdot l \cdot v^2}{d} \rightarrow v = \sqrt{\frac{h_d \cdot d \cdot 2g}{f \cdot l}}$$

$$Re \cdot \sqrt{f} = \sqrt{\frac{h_d \cdot d \cdot 2g}{l \cdot v^2}} \cdot \frac{v \cdot d}{v}$$

$$\frac{\text{kgf}}{\text{cm}^2} \cdot \frac{10^4 \text{ cm}^2}{\text{m}^2}$$

$$Re \cdot \sqrt{f} = \sqrt{\frac{0,5 \cdot 0,2 \cdot 19,62}{10}} \cdot \frac{0,2}{0,7 \cdot 10^{-6}}$$

(5.)

$$Re \cdot \sqrt{f} = 4,5 \cdot 10^4$$

$$\frac{D}{E} = \frac{0,1}{2,59 \cdot 10^{-4}} \rightarrow \frac{D}{E} = 386 \approx 400$$

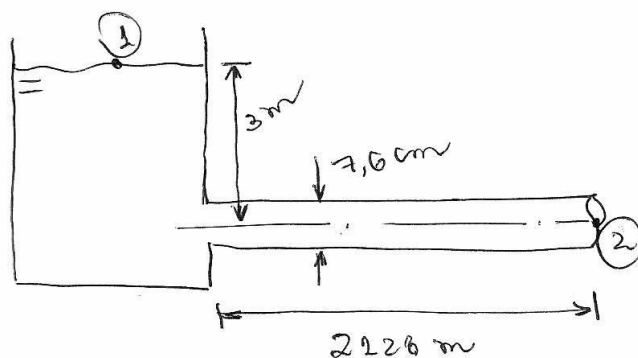
Diagrama Rouse - Hardy:  $f \approx 0,027$

$$v = \sqrt{\frac{0,5 \cdot 0,1 \cdot 19,62}{0,027 \cdot 10}} \rightarrow v = 1,91 \text{ m/s}$$

$$Q = A \cdot v = \pi \cdot \frac{0,1^2}{4} \cdot 1,91 \rightarrow Q = 15 \cdot 10^{-3} \text{ m}^3/\text{s}$$

- 9) Um grande reservatório de óleo tem um tubo de  $7,6 \text{ cm}$  e  $2126 \text{ m}$  de comprimento a ele conectado como mostrado na figura a seguir. A superfície livre do reservatório está a  $3 \text{ m}$  acima da linha do centro do tubo e pode ser considerada fixa nesta elevação. Admitindo escoamento laminar, calcule a velocidade média de escoamento e posteriormente verifique o n.º de Reynolds.

Dado:  $\nu_{\text{óleo}} = 0,2 \cdot 10^{-6} \text{ m}^2/\text{s}$



(G.)

sol: Bernoulli entre ① e ②:

$$\frac{P_1}{\rho} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho} + z_2 + \frac{v_2^2}{2g} + h_{p1 \rightarrow 2}$$

$$\left\{ \begin{array}{l} z_2 = 0 \text{ (NR)} \\ v_1 \gg v_2 \rightarrow v_1 \approx 0 \end{array} \right.$$

$$P_1 = P_2 = P_{atm}$$

$$z_1 = \frac{v_2^2}{2g} + h_d$$

$$h_d = f \cdot \frac{l}{d} \cdot \frac{v_2^2}{2g}$$

Admitindo fluxo laminar:  $f = \frac{64}{Re}$

$$h_d = \frac{64 \cdot \nu}{v_2 \cdot d} \cdot \frac{l}{d} \cdot \frac{v_2^2}{2g} = \frac{64 \cdot 9,2 \cdot 10^{-6}}{v_2 \cdot 0,076} \cdot \frac{2,128}{0,076} \cdot \frac{v_2^2}{2 \cdot 9,81}$$

$$h_d = 11,06 v_2$$

$$13 = \frac{v_2^2}{2 \cdot 9,81} + 11,06 v_2$$

$$0,05 v_2^2 + 11,06 v_2 - 13 = 0$$

$$v_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-11,06 \pm \sqrt{11,06^2 - 4 \cdot 0,05(-13)}}{2 \cdot 0,05}$$

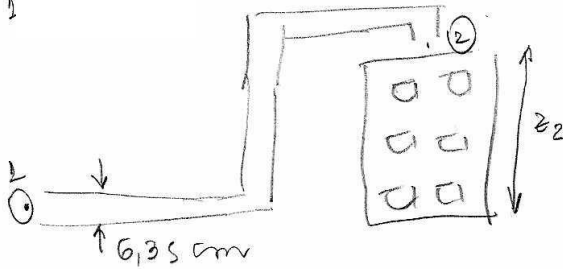
$$v_2 = \frac{-11,06 \pm 11,09}{0,1} \rightarrow \boxed{v_2 = 0,3 \text{ m/s}}$$

$v_2 = \cancel{22,1 \text{ m/s}}$

$$Re = \frac{v \cdot d}{\nu} = \frac{0,3 \cdot 0,076}{9,2 \cdot 10^{-6}} \rightarrow \boxed{Re = 2478}$$

Podem ser considerados laminar! (7.)

10)



$$v = 1,32 \cdot 10^{-6} \text{ m/s}$$

$$l = 152 \text{ m}$$

$$Q = 9,46 \text{ L/s}$$

$$z_2 = 26 \text{ cm}$$

$$P_1 = 7 \text{ kgf/cm}^2 = 664634,1 \text{ Pa}$$

tubo liso

sol: Bernoulli entre 1 e 2:

$$\frac{P_1}{\rho} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho} + z_2 + \frac{v_2^2}{2g} + h_{p1 \rightarrow 2}$$

$$z_1 = 0 \text{ (NR)}$$

$$A_1 \cdot v_1 = A_2 \cdot v_2 \rightarrow A_1 = A_2 \rightarrow v_1 = v_2$$

$$h_{p1 \rightarrow 2} = h_d \rightarrow \text{pis } h_d = 0 \text{ (disprezível)} \quad 0,5$$

$$\frac{9810 \text{ N}}{9810 \text{ kgf}} \cdot \frac{7 \text{ kgf/cm}^2}{9810} = \frac{P_2}{9810} + 26 + f \cdot \frac{l}{d} \cdot \frac{v^2}{2g}$$

$$h_d = f \cdot \frac{l}{d} \cdot \frac{v^2}{2g} = f \cdot \frac{152}{0,0635}$$

$$Q = A \cdot v \Rightarrow v = \frac{9,46 \text{ dm}^3/\text{s} \cdot 4 \cdot \frac{1 \text{ m}^3}{10^3 \text{ dm}^3}}{\pi \cdot (0,0635)^2} \rightarrow v = 2,99 \text{ m/s} \quad 0,5$$

$$Re = \frac{v \cdot d}{\nu} = \frac{2,99 \cdot 0,0635}{1,32 \cdot 10^{-6}} \Rightarrow Re = 1,4 \cdot 10^5 \text{ (turbulenta)} \quad 0,5$$

$$f = \frac{0,316}{Re^{0,25}} = \frac{0,316}{(1,4 \cdot 10^5)^{0,25}} \rightarrow f = 0,026 \quad 0,5$$

$$h_d = 0,026 \cdot \frac{152}{0,0635} \cdot \frac{2,99^2}{2 \cdot 9,81} \rightarrow h_d = 17,45 \text{ m} \quad 0,5$$

$$\frac{9810 \text{ N}}{9810 \text{ kgf}} \cdot \frac{9,006775 \text{ N/cm}^2}{9810} = \frac{P_2}{9810} + 26 + 17,45$$

$$P_2 = 238383,0 \text{ Pa} \quad 0,5$$

11)  $P_B = ?$   
man

$\epsilon = 0,15 \cdot 10^{-3} \text{ m}$

$Q = 40 \text{ L/s}$

$d_s = 15 \text{ cm}$

$d_R = 20 \text{ cm}$

$\nu = 20^{-6} \text{ m}^2/\text{s}$

$\gamma = 10^3 \text{ kgf/m}^3$

$$\left\{ \begin{array}{l} K_1 = 1,5 \\ K_2 = K_4 = K_7 = 0,9 \\ K_3 = K_6 = 1,0 \\ K_5 = 0,5 \\ K_8 = 1,0 \end{array} \right.$$

Sol:  $\frac{P_0}{\gamma} + \frac{v_0^2}{2g} + z_0 = \frac{P_a}{\gamma} + \frac{v_a^2}{2g} + z_a + h_{p_0 \rightarrow a}$

$$\left\{ \begin{array}{l} P_1 = P_{atm} = 0 \text{ (P_{atm})} \\ v_0 = v_a = 0 \\ z_0 = 0 \text{ (NR)} \end{array} \right. \quad 0,5 \quad h_{p_0 \rightarrow a} = h_d + h_l$$

$P_a = (z_a + h_{p_0 \rightarrow a}) \gamma$

Como os diâmetros são diferentes divide-se em 2 partes:  
 $d_s = 0,15 \text{ m}$

• Ds:  $h_p = \left( f \cdot \frac{l}{d_s} + \sum K \right) \frac{v_s^2}{2g}$

$Q = A \cdot v_s \rightarrow v_s = \frac{40 \cdot 10^{-3}}{\pi \cdot 0,15^2} \rightarrow v_s = 2,26 \text{ m/s}$

$\sum K = 0,5 + 1,0 + 0,9 + 1,0 = 3,4$

$f = ? \rightarrow Re = \frac{v \cdot d_s}{\nu} = \frac{2,26 \cdot 0,15}{10^{-6}} \rightarrow Re = 3,4 \cdot 10^5$   
 $\frac{d_s}{\epsilon} = \frac{0,15}{0,15 \cdot 10^{-3}} \rightarrow \frac{d_s}{\epsilon} = 1000$   
 D. de Colebrook  
 $f = 0,021$

$h_{pds} = \left( 0,021 \cdot \frac{36}{0,15} + 3,4 \right) \cdot \frac{2,26^2}{19,62} \rightarrow h_{pds} = 4,54 \text{ m}$

$$DR: \quad h_p = \left( f \cdot \frac{l}{d} + \epsilon K \right) \frac{v_R^2}{2g}$$

$$Q = A \cdot v_R \rightarrow v_R = \frac{40 \cdot 10^{-3} \cdot 4}{\pi \cdot 0,10^2} \rightarrow v_R = 5,1 \text{ m/s} \quad 0,25$$

$$\epsilon K = 15 + 9,9 + 10 + 9,9 = 26,8$$

$$f = ? \quad \left\{ \begin{array}{l} Re = \frac{v \cdot d}{\nu} = \frac{5,1 \cdot 0,10}{10^{-6}} \rightarrow Re = 5,1 \cdot 10^5 \\ \frac{d\epsilon}{\epsilon} = \frac{0,10}{9,15 \cdot 10^{-3}} \rightarrow \frac{d\epsilon}{\epsilon} = 600,66 \approx 700 \end{array} \right. \quad \left\{ \begin{array}{l} \text{diagrama} \\ f = 0,0225 \\ 0,5 \end{array} \right.$$

$$h_{pde} = \left( 0,0225 \cdot \frac{40,5}{0,1} + 26,8 \right) \cdot \frac{5,1^2}{19,62} \rightarrow h_{pde} = 38,66 \text{ m} \quad 0,25$$

$$h_{p0 \rightarrow 0} = 4,54 + 38,66 \rightarrow h_{p0 \rightarrow 0} = 43,2 \text{ m} \quad 0,25$$

$$P_0 = -(6,5 + 43,2) \cdot 10^3 \text{ kgf/m}^2$$

$$\boxed{P_0 = -4,9 \cdot 10^4 \text{ kgf/m}^2} \quad 0,5$$